Topic: Increasing and decreasing

Question: Determine the intervals on which the function is increasing and decreasing.

$$f(x) = x^4 - 3x^2$$

Answer choices:

A Increasing on $\left(-\sqrt{3},0\right)$ and $\left(\sqrt{3},\infty\right)$

Decreasing on $\left(-\infty,-\sqrt{3}\right)$ and $\left(0,\sqrt{3}\right)$

B Increasing on $\left(-\sqrt{\frac{3}{2}},0\right)$ and $\left(\sqrt{\frac{3}{2}},\infty\right)$

Decreasing on $\left(-\infty,-\sqrt{\frac{3}{2}}\right)$ and $\left(0,\sqrt{\frac{3}{2}}\right)$

C Increasing on $\left(\sqrt{\frac{1}{2}},0\right)$

Decreasing on $\left(-\infty,-\sqrt{\frac{1}{2}}\right)$

D Increasing on $\left(-\infty,-\sqrt{3}\right)$ and $\left(\sqrt{3},\infty\right)$

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Decreasing on $\left(-\sqrt{3},0\right)$ and $\left(0,\sqrt{3}\right)$

Solution: B

A function is increasing on intervals where the derivative of the function is positive and decreasing on intervals where the derivative of the function is negative, and makes the transition at critical points, where the derivative of the function is 0. So, we first find the critical numbers by setting the first derivative of the function equal to 0 and solving for x.

$$f'(x) = 4x^3 - 6x$$

$$4x^3 - 6x = 0$$

$$2x\left(2x^2 - 3\right) = 0$$

$$2x = 0$$

$$x = 0$$

or

$$2x^2 - 3 = 0$$

$$x^2 = \frac{3}{2}$$

$$x = \pm\sqrt{\frac{3}{2}}$$

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We use these critical numbers to create intervals on which to check the value of the derivative.

Interval	$\left(-\infty, -\sqrt{3/2}\right)$	$\left(-\sqrt{3/2}, 0\right)$	$\left(0, \sqrt{3/2}\right)$	$\left(\sqrt{3/2}, \infty\right)$
Test value	$x = -2$	$x = -1$	$x = 1$	$x = 2$
$f'(x)$	$4(-2)^3 - 6(-2)$	$4(-1)^3 - 6(-1)$	$4(1)^3 - 6(1)$	$4(2)^3 - 6(2)$
	-20, Negative	2, Positive	-2, Negative	20, Positive
$f(x)$	Decreasing	Increasing	Decreasing	Increasing

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Topic: Increasing and decreasing

Question: Where is the function increasing and decreasing?

$$f(x) = x^2$$

Answer choices:

A Increasing when $x < 1$ and decreasing when $x > 1$

B Increasing when $x < 0$ and decreasing when $x > 0$

C Increasing when $x > 0$ and decreasing when $x < 0$

D Increasing when $x > 1$ and decreasing when $x < 1$

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Solution: C

To find the critical numbers, take the first derivative of the function

$$f(x) = x^2$$

$$f'(x) = 2x$$

Now set the derivative equal to 0 and solve for x.

$$0 = 2x$$

$$x = 0$$

There is one critical number at $x = 0$. Now take a point on either side of $x = 0$ (we will use $x = -1$ and $x = 1$) and substitute these into the first derivative.

$$f'(-1) = 2(-1)$$

$$f'(-1) = -2$$

and

$$f'(1) = 2(1)$$

$$f'(1) = 2$$

We can see that on the left-hand side of $x = 0$ the function is negative, so it's decreasing. On the right-hand side of $x = 0$ the function is positive, so it's increasing.

The function $f(x) = x^2$ is increasing when $x > 0$ and is decreasing when $x < 0$.

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Topic: Increasing and decreasing

Question: Where is the function increasing and decreasing?

$$f(x) = x^4 - 4x^3 + 4x^2 - 7$$

Answer choices:

A Decreasing when $x < 0$, increasing between $x = 0$ and $x = 1/2$, decreasing between $x = 1/2$ and $x = 3/2$, and increasing when $x > 3/2$.

B Decreasing when $x < 0$, increasing between $x = 0$ and $x = 1$, decreasing between $x = 1$ and $x = 2$, and increasing when $x > 2$.

C Increasing when $x < 0$, decreasing between $x = 0$ and $x = 1/2$, increasing between $x = 1/2$ and $x = 3/2$, and decreasing when $x > 3/2$.

D Increasing when $x < 0$, decreasing between $x = 0$ and $x = 1$, increasing between $x = 1$ and $x = 2$, and decreasing when $x > 2$.

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Solution: B

To find the critical numbers, take the first derivative of the function

$$f(x) = x^4 - 4x^3 + 4x^2 - 7$$

$$f'(x) = 4x^3 - 12x^2 + 8x$$

$$f'(x) = 4x\left(x^2 - 3x + 2\right)$$

$$f'(x) = 4x(x - 2)(x - 1)$$

Now set the derivative equal to 0 and solve for x.

$$0 = 4x(x - 2)(x - 1)$$

$$x = 0,\ 1,\ 2$$

There are three critical numbers at $x = 0,\ 1,\ 2$. Now take a point in each interval (we will use $x = -1$, $x = 1/2$, $x = 3/2$, and $x = 3$) and substitute these into the first derivative.

$$f'(-1) = 4(-1)^3 - 12(-1)^2 + 8(-1)$$

$$f'(-1) = -4 - 12 - 8$$

$$f'(-1) = -24$$

and

$$f'\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right)$$

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$$f'\left(\frac{1}{2}\right) = 4\left(\frac{1}{8}\right) - 12\left(\frac{1}{4}\right) + 4$$

$$f'\left(\frac{1}{2}\right) = \frac{1}{2} - 3 + 4$$

$$f'\left(\frac{1}{2}\right) = \frac{3}{2}$$

and

$$f'\left(\frac{3}{2}\right) = 4\left(\frac{3}{2}\right)^3 - 12\left(\frac{3}{2}\right)^2 + 8\left(\frac{3}{2}\right)$$

$$f'\left(\frac{3}{2}\right) = 4\left(\frac{27}{8}\right) - 12\left(\frac{9}{4}\right) + 12$$

$$f'\left(\frac{3}{2}\right) = \frac{27}{2} - 27 + 12$$

$$f'\left(\frac{3}{2}\right) = -\frac{3}{2}$$

and

$$f'(3) = 4(3)^3 - 12(3)^2 + 8(3)$$

$$f'(3) = 4(27) - 12(9) + 24$$

$$f'(3) = 108 - 108 + 24$$

$$f'(3) = 24$$

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We can see that on the left-hand side of $x = 0$ the function is negative, so it's decreasing. Between $x = 0$ and $x = 1$, the function is positive, so it's increasing. Between $x = 1$ and $x = 2$, the function is negative, so it's decreasing. On the right-hand side of $x = 2$ the function is positive, so it's increasing.

The function $f(x) = x^4 - 4x^3 + 4x^2 - 7$ is decreasing when $x < 0$, increasing between $x = 0$ and $x = 1$, decreasing between $x = 1$ and $x = 2$, and increasing when $x > 2$.

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